Engineering Note

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Use of Short Period Frequency Requirements in Horizontal Tail Sizing

David J. Moorhouse* and Michael W. M. Jenkins†
Lockheed-Georgia Company, Marietta, Ga.

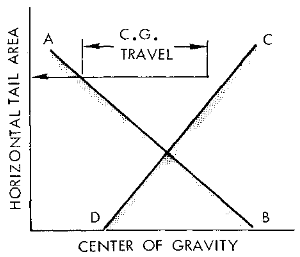

Fig. 1 Typical horizontal tail sizing diagram.

I. Introduction

IN establishing the preliminary design configuration of an aircraft concept, one of the first tasks is to size the empennage. The horizontal tail is sized by considerations of trim, stability and the required center of gravity travel during flight, as illustrated in Fig. 1. The line AB represents the forward limit on center of gravity from controllability considerations such as trim or nose wheel lift-off. The line CD is the locus of points of neutral static stability and is the basis of the aft center of gravity limit. With some margin from the neutral stability line, the required center of gravity travel then determines the required horizontal tail area. However, there is no specified requirement for any particular value of the margin from the neutral stability line, or minimum static margin. The flying qualities requirement of Ref. 1 is in terms of short period frequency and normal acceleration response. For preliminary design applications an arbitrary minimum static margin is usually assumed. The longitudinal dynamic stability is evaluated later and if the specified criteria are not met, then a design iteration is created. The benefits of using specified requirements as a design tool are obvious, and the objective of this Note is to present such a technique.

II. Analysis

Ref. 1 gives maximum and minimum values of $\omega_{n_{sp}}^2/n_\alpha$ as the short period frequency criteria for $\omega_{n_{sp}} > 1.0$. An approximate solution to the equations of motion gives

$$\omega_{n_{sp}}^2 \simeq -\frac{q S \bar{c}}{I_y}\left[\frac{\rho S \bar{c}}{4m}\cdot C_{L_\alpha}\cdot C_{m_q} + C_{m_\alpha}\right]$$

Also,

$$n_\alpha \simeq (qSC_{L_\alpha}/W)$$

Therefore,

$$\frac{\omega_{n_{sp}}^2}{n_\alpha} \simeq -\frac{W\bar{c}}{I_y}\left[\frac{\rho S\bar{c}}{5m}\cdot C_{m_q} + C_{m_{C_L}}\right]$$

The pitch damping coefficient can be approximated by

$$C_{m_q} \simeq -2.2\,\frac{S_H}{S}\cdot\frac{\mathcal{L}_T^2}{\bar{c}^2}\,(C_{L_\alpha})_H$$

and by definition

$$-C_{m_{C_L}} = \text{static margin, h}$$

therefore

$$\frac{\omega_{n_{sp}}^2}{n_\alpha} = \frac{1.1\, g\, \rho\, S_H \mathcal{L}_T^2}{2I_y}\,(C_{L_\alpha})_H + \frac{W\bar{c}}{I_y}\,h \qquad (1)$$

The term $\omega_{n_{sp}}^2/n_\alpha$ is known as the control anticipation parameter, C.A.P., and Eq. (1) shows that this parameter increases with increasing tail area and increasing static margin, and is largely independent of the flight phase, with altitude appearing through the density term. The other terms in Eq. (1) are gross weight, moment of inertia, mean aerodynamic chord, tail arm, and planform (lift curve slope) of the horizontal tail which may all be considered constants of the configuration.

To use this result in sizing the horizontal tail for the minimum area, Eq. (1) is rearranged to give

$$S_H > K_I\,[\text{CAP}_{\min} - K_2 h_{\min}]$$

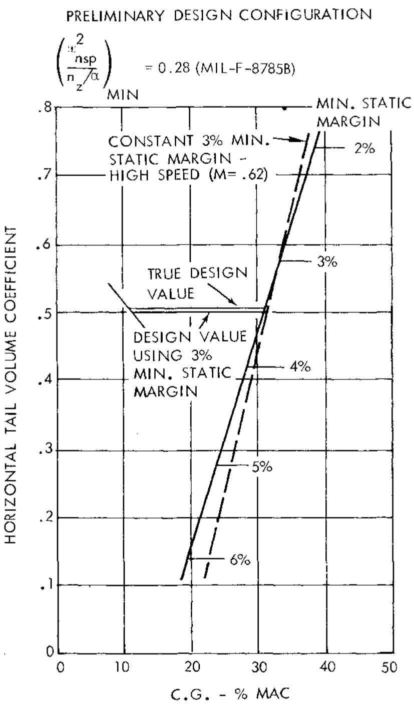

Fig. 2 Effect of minimum short period response on tail.

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*Presently Aerospace Engineer, USAF Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio.
†Aircraft Development Engineer-Specialist. Member AIAA.

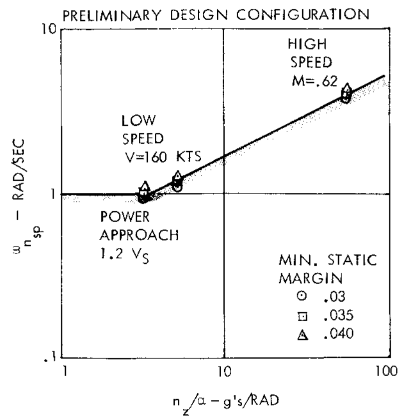

Fig. 3 Short period frequency vs static margin.

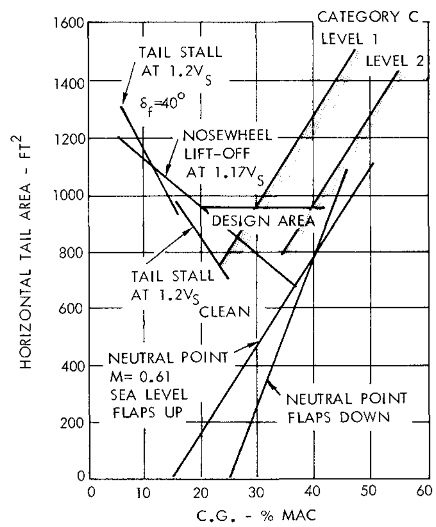

Fig. 4 Effect of minimum short period response on C-5A horizontal tail requirement.

where

$$K_1 = \frac{2ly}{1.1\, g\rho \mathcal{S} \bar{c}\, (C_{l_\alpha})_H} \qquad K_2 = \frac{W\bar{c}}{I_y}$$

Reference 1 gives different values of the minimum $\omega n_{sp}^2/n_\alpha$ depending on the flight category; however, the relevant one is the largest one, i.e. 0.28 for Level 1, Category A. The smaller required values of CAP are then exceeded automatically. It is also recognized that just meeting this minimum requirement does not guarantee acceptable flying qualities. It is believed to be adequate for preliminary design use, however.

Figure 2 shows the use of the suggested procedure on a fighter configuration needing a 20% C.G. range compared with using an arbitrary 3% minimum static margin. For this design there is only a small change in required tail volume coefficient, but the minimum design static margin is indicated immediately as 3.5% means aerodynamic chord. Figure 3 represents the results of a full three-degree-of-freedom calculation of longitudinal dynamic stability for the same configuration. Static margin was varied for power approach, low speed, and high speed conditions. The results show that in order to meet the minimum frequency criterion a minimum static margin of 3.4 to 3.9% is required throughout the speed range. Although another criterion is obviously required for the region where $\omega_{n_{sp}} > 1$, these results are in excellent agreement with the simple procedure proposed in this Note.

By contrast, Fig. 4 shows the results of the procedure applied to the C-5A configuration. Category A of Ref. 1 is assumed not to apply to this type of aircraft, the appropriate criteria are those of Category C ($\omega_{n_{sp}}^2/n_\alpha$ of 0.16 for Level1 and 0.096 for Level 2). It would obviously be impractical to meet the Level 1 requirements without a stability augmentation. Thus, in the earliest design phase, the procedure indicates a requirement for, and probable level of, a stability augmentation system.

III. Conclusions

A procedure is suggested for determining the aft center of gravity limit required to meet the flying qualities specification for short period dynamics. It is recognized that the procedure does not necessarily guarantee acceptable flying qualities. It does form a rational method for calculating the aft center of gravity limit in the initial design phase of sizing the horizontal tail of a new airplane configuration. For configurations where this criterion is unduly restrictive, the developed procedure immediately indicates the need for a stability augmentation system.

References

[1] Anon., "Military Specifications. Flying Qualites of Piloted Airplanes," MIL-F-8785B (ASG), Aug. 1969.

Technical Comments

Comment on "Solution of the Lifting Line Equation for Twisted Elliptic Wings"

Rajendra K. Bera*
National Aeronautical Laboratory, Bangalore, India

FILOTAS[1] has given an explicit solution for the case of a sinusoidally twisted elliptic wing using the Prandtl lifting line equation in which the circulation distribution is expanded in an infinite series of Chebyshev polynomials of the second kind. The intention of this Note is to show that a simpler explicit solution exists for the conventional sine series expansion for the circulation, Γ

$$\Gamma(\theta) = \sum_{n=1}^{\infty} A_n \Gamma_n(\theta) = 2sU \sum_{n=1}^{\infty} A_n \sin n\theta \tag{1}$$

where s is the wing semispan, U the freestream velocity, A_n are constants, $\theta = \cos^{-1}(y/s)$ measured from wing tip to tip, and y is the spanwise coordinate positive on the right wing.

In Ref. 2, an integral form of the solution is derived for the general case of a twisted wing which reads [Ref. 2, Eq. (12)]

$$\frac{\pi}{2} sUnA_n + (1/\pi U) \int_0^\pi [\Gamma_n(\theta) \sum_{r=1}^{\infty} A_r \Gamma_r(\theta)/c(\theta)] \sin\theta \, d\theta = \int_0^\pi \alpha(\theta) \Gamma_n(\theta) \sin\theta \, d\theta \tag{2}$$

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*Scientist, Aerodynamics Division.